B.Sc. Part I

Paper I

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Diary

Q6 What is Doppler effect? Discuss relativi-
stically the phenomenon of Doppler effect
light and mention its applications in
Astrophysics

Ans **Doppler effect:**

An apparent change in frequency of received radiation due to relative motion between source, observer medium is known as Doppler effect.

Let us consider a plane wave whose wave normal points in direction gi- by direction cosines l, m and n w.r.t. a- observer in the frame S. Let its phase velo- be u such that its components are ul, um, ~~and~~ un and let ~~its~~ its wave length, frequency & time Period be λ, ν an τ respectively such that $\lambda = u/\nu = u\tau$.

These quantities in frame S' are written with Primed Symbols. Let us take an equation of a plane wave as

$$\psi = A\exp.\left[2\pi i\left(\frac{lx+my+nz}{\lambda} - \frac{t}{\tau}\right)\right] \quad \text{--- (1)}$$

$$= A\exp.\left[\vec{k}\cdot\vec{x} - \frac{2\pi i t}{\tau}\right] \quad \text{--- (2)}$$

where vector $\vec{k} = \frac{2\pi}{\lambda}(l, m, n)$ and points in the direction of wave propagation and $\vec{x} = (x, y, z)$.

Now let us introduce a set of four quantities $(k_i) = \left(\vec{k}\cdot\frac{i\omega}{c}\right)$

where $\omega = \frac{2\pi}{\tau} = 2\pi\nu$. with This introduction equn (1) becomes

$$\psi = A\exp.(k_i, x_i) \quad \text{--- (3)}$$

where (x_i) is a four Position vector, as defined subsequently in four-vectors. The inner Product of (k_i) with four vector (x_i) gives a scalar quantity, hence (k_i) is also a four vector with following usual spatial and temporal transformations as given in derivation of Lorentz transformation relations.

$$\therefore \vec{K'} = \vec{K} + \frac{(\vec{K}\cdot\vec{V})\vec{V}}{V^2}(\beta-1) - \beta\vec{V}\frac{\omega}{c^2} \quad \text{--- (4)}$$

and

$$\omega' = \beta[\omega - (\vec{K}\cdot\vec{V})] \quad \text{--- (5)}$$

where $\beta = \dfrac{1}{\sqrt{1-v^2/c^2}}$

eqn (1) (3) and (4) gives the direction of propagation in frame S and S' while eqn (5) inter-relates the frequencies in S and S'.

Since we have $\vec{K} = \dfrac{2\pi}{\lambda}\dfrac{\vec{u}}{u} = 2\pi\nu\dfrac{\vec{u}}{u^2}$

Therefore eqn (5) gives

$$\omega' = 2\pi\nu' = \beta\left[2\pi\nu - 2\pi\nu\frac{(\vec{u}\cdot\vec{V})}{u^2}\right]$$

or

$$\boxed{\nu' = \beta\nu\left[1 - \frac{\vec{u}\cdot\vec{v}}{u^2}\right]} \quad \text{--- (6)}$$

This gives the relativistic Doppler effect and it different from classical — due to the factor

$$\beta = \frac{1}{\sqrt{1-v^2/c^2}}$$

If we apply it to the case of light propagating in free space. we have

$= c$, and for the source system attached to the frame S', we have $\nu' = \nu_0$

Hence eqn (6) gives

$$\nu_0 = \nu\beta\left[1 - \frac{\vec{c}\cdot\vec{v}}{c^2}\right]$$

or

$$\nu = \frac{\nu_0\sqrt{1-v^2/c^2}}{\left(1 - \dfrac{\vec{c}\cdot\vec{v}}{c^2}\right)} \quad \text{--- (7)}$$

∴ In non-relativistic case where $\frac{v^2}{c^2} \ll 1$, we get

$$\boxed{\nu = \frac{\nu_0}{\left(1 - \frac{\vec{c}\cdot\vec{v}}{c^2}\right)}} \quad \text{---} \quad (8)$$

This is the usual classical relation. The following cases of interest may be studied with the help of equ^n (7).

⟨i⟩ In case the source of light moves radially away from the observer, we have $\vec{c}\cdot\vec{v} = -cv$. In this case relation in equ^n (7) becomes

$$\nu = \nu_0 \sqrt{\frac{c-v}{c+v}} \quad \text{---} \quad (9)$$

which shows that $\nu_0 \gg \nu$. However in classical case in which $v^2/c^2 \ll 1$, we get from equ^n (7) $\nu = \nu_0\left[\frac{c}{c+v}\right]$ — (10)

⟨ii⟩ In alternate case in which the source moves radially towards the observer, we have $\vec{c}\cdot\vec{v} = cv$ and equ^n (7) becomes

$$\nu = \nu_0 \sqrt{\frac{c+v}{c-v}} \quad \text{---} \quad (11)$$

which shows that $\nu > \nu_0$ in such a case. However in classical case when $\frac{v^2}{c^2} \ll 1$, we get from equ^n (7)

$$\nu = \nu_0\left[\frac{c}{c-v}\right] \quad \text{---} \quad (12)$$

(iii)

In case when the source moves at right angle to the observer $\vec{n} \cdot \vec{v} = 0$ and equⁿ (7) becomes

$$v = v_0 \sqrt{1 - \frac{v^2}{c^2}} \qquad (13)$$

~~This source~~

This shows that no transverse Doppler effect exists in classical case while it exists in relativistic case only with $v_0 > v$.